Raising Social Capital:
Tokenizing a Customer-Driven Business

An Introduction to Discount Token Economics

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This paper provides a brief formal specification of the *discount token* cryptoeconomic framework that was initially proposed by founders of Sweetbridge and subsequently extended to the general model presented here. Discount tokens enable customer-driven business models. They empower users to receive additional long-term value as effective co-owners, while diminishing the role of passive investors. Mediated by the blockchain, discount tokens enable organizations that are fair, customer-oriented, and long-term sustainable. Such organizations are akin to decentralized cooperatives, but with clear growth incentives of for-profit businesses.

1 Introduction

In the last six months, more than a billion dollars worth of capital has been invested in decentralized software projects via ICOs. The flood of capital, limited amount of functional software delivered, and the ensuing erosion of trust have fostered questions regarding the viability of blockchain-based organizational models. Specific concerns expressed by regulators, insiders, investors, and external observers focus on token models that (1) resemble securities in that they promise returns on passive investments, or (2) direct value from late investors to early investors and prompt comparisons with Ponzi schemes.

At the same time, it is clear that blockchain-based economic models offer an infinite design space with respect to incentive models within innovative organizational structures, and so responsible experimentation with token economics should be encouraged. As creators who desire to be both responsible and compliant, we find ourselves in search of a class of token economics that would (1) underlie a broad range of decentralized organizations and networks; (2) align incentives between investors (both early and late), creators, and consumers; (3) be demonstrably distinct from securities and Ponzi structures in the incentives they generate; and (4) align with existing regulatory precedent.

The discount token framework is designed to meet these requirements; while simple at its core, it has profound implications. In a discount token economy, creators and users of the network are clearly aligned, while passive investors and speculators find themselves at an economic disadvantage. This is because discount tokens are more economically valuable to users than passive investors and will discourage unhealthy price dynamics prevalent in other classes of cryptoassets.

The discount token model is applicable to a broad range of business models. Businesses that most benefit from it are those that (1) require a significant investment in development of intellectual property or asset purchases, and (2) desire and expect significant long-term business from its customers through ongoing subscriptions, recurrent fees, or frequent repeat purchases. In this document a software-as-a-service (SaaS) business is chosen as a representative of this class to illustrate the application of discount token economics.

As applied to an SaaS business, the tokens capture a share in the active use of the software, and thus their value is drawn from and realized by the use of it. A discount token is well-defined, even when the software provided is itself centralized and is thus particularly suited to crowdfunding projects the MVP of which consists of a centralized version of an application being iteratively decentralized. The authors believe that this a very practical approach to transitioning the backend of our economy into a more efficient, decentralized state.

This document is an introduction to the mathematics of discount tokens and briefly touches on its implications; an extensive discussion of a discount token in the Sweetbridge liquidity protocol, Sweetcoin, can be found in [31]. The Sweetbridge ecosystem will provide explicit support for discount token issuance and distribution as part of the Sweetbridge Crowdsale Platform, [18]. The work presented builds upon a foundation of rigorous mathematical work across the fields of engineering, economics and psychology with attention to coordination, fairness, incentives and decisions in all areas of study. See Section 4 for more details and direct references to academic and industry research.

The authors hope that the introduction of a straightforward discount token archetype will foster responsible decentralized product development efforts, clarify regulatory concerns, drive greater transparency, and create better incentive alignments as compared to the older frameworks.

2 Mechanics of Discount Tokens

In brief, discount tokens are digital assets that give their holders a limited right to receive discounts on purchases of products or services from an organization – a company, a coop, or a blockchain network. For brevity, considering that this model is most pertinent to blockchain-based services, we will refer to any organization as “network,” with the understanding that the spectrum of service providers who can use this model is much broader. Similarly, we will use the words “users” and “customers” interchangeably.

Unlike gift cards, discount tokens are not invalidated when used (“burned” in blockchain parlance), but remain in possession of the holders. The specific size of the discount that each token realizes for its owner is designed to grow in step with the overall utilization of the network. However, the maximum discount customers can receive is limited to a given percentage, which could sometimes be as high as 100%, making services effectively free for some. Notwithstanding the fact that some users may receive free services, the discount token model ensures that the total discounts networkwide never reach 100%, and thus the network always has sufficient funding to operate.

Let’s consider an example.

BlockChainMail is a blockchain-based SaaS business. Their product is a software platform for Renaissance faire participants to trade or loan costumes and props to one-another while tracking ownership rights and current possession on the blockchain. The software being built provides a service that has a baseline value c USD per month to end users under a traditional SaaS model.

BlockChainMail implements their token, designated BCM, to be a software license by defining the discount model:

$$C(t) = \begin{cases} t \le \frac{T}{U} & c \cdot (1 - \frac{t \cdot U}{T}) \\ \text{else} & 0 \end{cases} , \tag{2.1}$$

where $C(t)$ is the fiat cost paid by a user per specified time period when they activate t BMC tokens in the product’s smart contract; U is the total number of users currently subscribed to the service; and T is the total number of BMC tokens currently activated.

Putting specific numbers to this model, let’s assume (1) that the service costs \$10 per month; (2) that there are 1,000 users registered in the network, and (3) that 100,000 tokens were distributed to users, who are actively using them to access the service. In this situation, one user needs 100 tokens to receive services for free.

A user must take an extra step of activating their tokens in order to start receiving discounts per the above formula. As we will see in the course of this paper, this formulation ensures that the discount size increases as the network (as represented as the value U) grows.

The value $t_{max} = T/U$ is called the activation limit. Users are not able to activate more than t_{max} tokens. We can say that in this case $t_{max} = t_{free}$, the number of tokens that makes services free.

The total T only includes tokens that are actually activated and, consequently, is limited by actual network utilization, making it impossible for large holders who do not fully realize their discounts to diminish the capacity of the user's tokens to provide discounts.

In the course of this paper, we will examine the general form of the discount token economics and the incentives it creates. The rest of Chapter 2 presents the mathematical specification that addresses: (1) calculating the fees in the presence of discount tokens; (2) valid discount models; (3) network utilization metrics that determine the discount dynamics, ensuring positive behavior under the growth scenario; (4) network limits that ensure sufficient operating revenue in the presence of discount tokens; and (5) the economic components of the token value.

2.1 General Formulation of Discount Token Economics

Given the price per unit of service c, we define the cost of services

$$C(t, y; X) = c \cdot y \cdot (1 - f(t, y; X)), \tag{2.2}$$

where t is the number of discount tokens activated, y is the quantity or level of service purchased by the user during the license period, and X denotes the global network state or any material characteristics thereof. The function $f(t, y; X)$ is the component of the model called the *discount function.* It is assumed that f is formulated in such a way that C is always nonnegative, that is, discounts never turn into profits. Consequently, it is assumed that there is a value $t_{max}(X)$, such that $t \leq t_{max}(X)$, which limits the ability of the user to activate tokens.

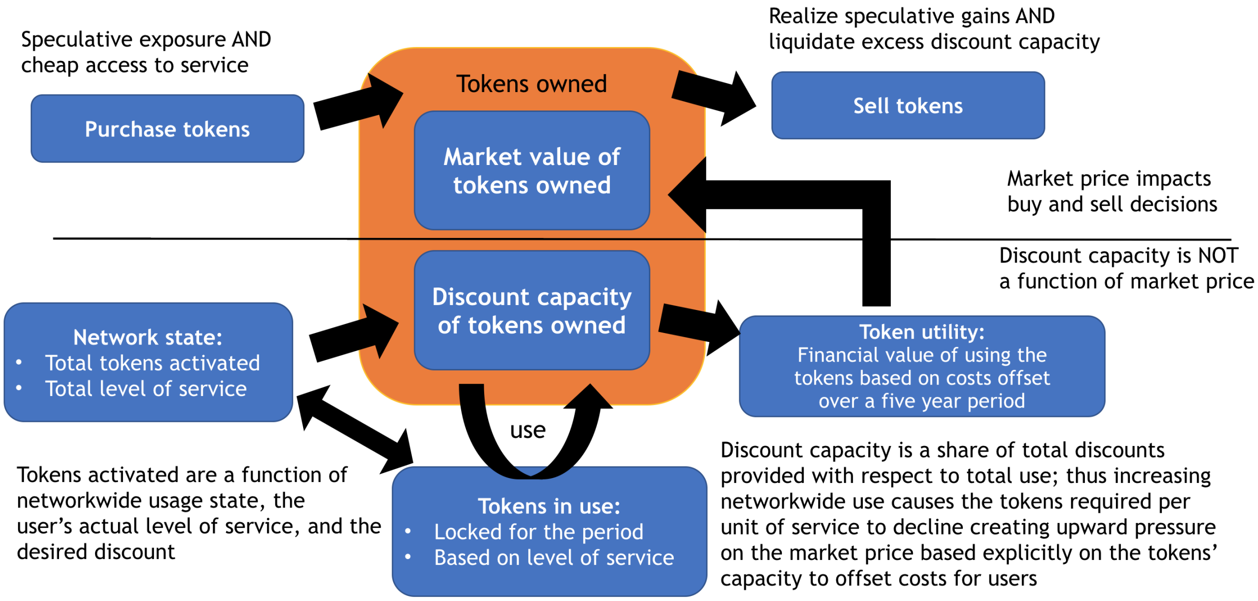

Figure 2.1: **Discount Token Lifecycle.** The use of a discount token unburdens the token from the role of currency and allows it to function directly as a software license; the mechanics of the token revolve around use and thus drive rather than get driven by the market price.

2.2 Basic Implications

In this model, the product is accessible to non-token-holders in exchange for fiat or cryptocurrency at prices set by the service provider or determined by the open market. Compare this to cryptoeconomic models in which the transaction currency used to purchase the service is specific to the given network. In such systems, the transaction currency is expected to appreciate with increased network utilization due to money supply dynamics. Unfortunately, tokens serving as product-specific currency have a number of drawbacks: (1) a transaction currency that appreciates or otherwise fluctuates in price disincentivizes its use to purchase services and makes provision of services an economically risky process – in this way, it behaves contrary to its stated goal; and (2) users must acquire and hold the specific currency to take advantage of the specific service, introducing frictions for non-savvy cryptocurrency holders and requiring significant support in the cryptocurrency exchange ecosystem. While this last point may not seem like a big deal, remember that the crypto-savvy community is still small, and mainstream adoption of products certainly depends on engagement by a broader class of users. In general, the requirement to use specific tokens to access specific services may introduce frictions that are contrary to the project's goals.

In contrast, the discount token model allows the best choice of transaction currency, as dictated by the needs of the network and nothing else. Any stable transaction currency, any pure volatile cryptocurrency such as ether or bitcoin, or any other form of economic value may be used to purchase services. The discount tokens act to provide incentives for users and early supporters without constraining the transaction economics of the network in the future.

2.3 Defining the Discount Function

In the case of BlockChainMail, the value y – number of service units purchased – is always 1 because the service is simply the right for one user to use the software. BlockChainMail defines their discount function as

$$f(t;X) = \frac{t \cdot X_U}{X_T} \tag{2.3}$$

and defines

$$t_{max}(X) = t_{free}(X) = \frac{X_T}{X_U} \tag{2.4}$$

We now list general criteria for valid discount functions.

Criterion 1. *The discount function* $f(t,y;X)$ *must have the property*

$$f(0,y;X) = 0 \tag{2.5}$$

for any level of service y*, and valid network state* X*; defining the trivial condition that activating no tokens generates no discount.*

Criterion 2. *The number of tokens required to eliminate all fees for* y *units of service is*

$$t_{free}(y;X) \tag{2.6}$$

which exists for any y *and* X*, and always satisfies*

$$f\left(t_{free}(y;X), y; X\right) = 1. \tag{2.7}$$

Defining the condition that there is a finite $t_{free}(y;X)$ *achieving full discount for any* y *and* X*.*

Criterion 3. *The number of tokens required for the 100% discount,* $t_{free}(y;X)$ *is strictly increasing in* y

$$\frac{\partial}{\partial y} t_{free}(y;X) > 0 \text{ for all } X. \tag{2.8}$$

This ensures that given any state of the network X*, the amount of tokens required to access the service for free is increasing with the level of use of the service characterized by* y*.*

Criterion 4. *The maximum number of tokens allowed for activation is bounded by*

$$t_{max}(y;X) \leq t_{free}(y;X). \tag{2.9}$$

This bound ensures that hoarding of tokens does not allow incentive manipulation by activating far more tokens than needed for the service actually used.

2.4 Activation Limits and Traction Metrics

Discount tokens are designed to reflect the overall usage of the network. The choice of traction metrics and the design of the function $f(t,y;X)$ will directly influence the token economics once the network is live with active users. Having defined $t_{free}(y;X)$, the simplest version of this solution is linear scaling per unit of service, which is exactly what we have in our example.

$$t_{free}(y;X) = y \cdot t_{free}(1;X). \tag{2.10}$$

which means that

$$\begin{aligned} f(t,y;X) &= \frac{t}{t_{free}(y;X)} && (2.11) \\ &= \frac{t}{y \cdot t_{free}(1;X)} && (2.12) \end{aligned}$$

where $t_{free}(1;X)$ designates a number of tokens that make a single unit of service free. Rewriting equation (2.2) based on this formulation, we get

$$\begin{aligned} C(t,y;X) &= c \cdot y \cdot \left(1 - \frac{t}{y \cdot t_{free}(1;X)}\right) && (2.13) \\ &= c \cdot \frac{y \cdot t_{free}(1;X) - t}{t_{free}(1;X)} && (2.14) \end{aligned}$$

valid for t in the range $0 \leq t \leq y \cdot t_{free}(1;X)$.

The goal here is to design $t_{free}(y;X)$ in such a way that t_{free} diminishes as the network gains traction, effectively allowing the token holder to realize the benefits of the growing network utilization directly through an increase in access, either by using more units of service or by sharing or selling the tokens, enabling even more use.

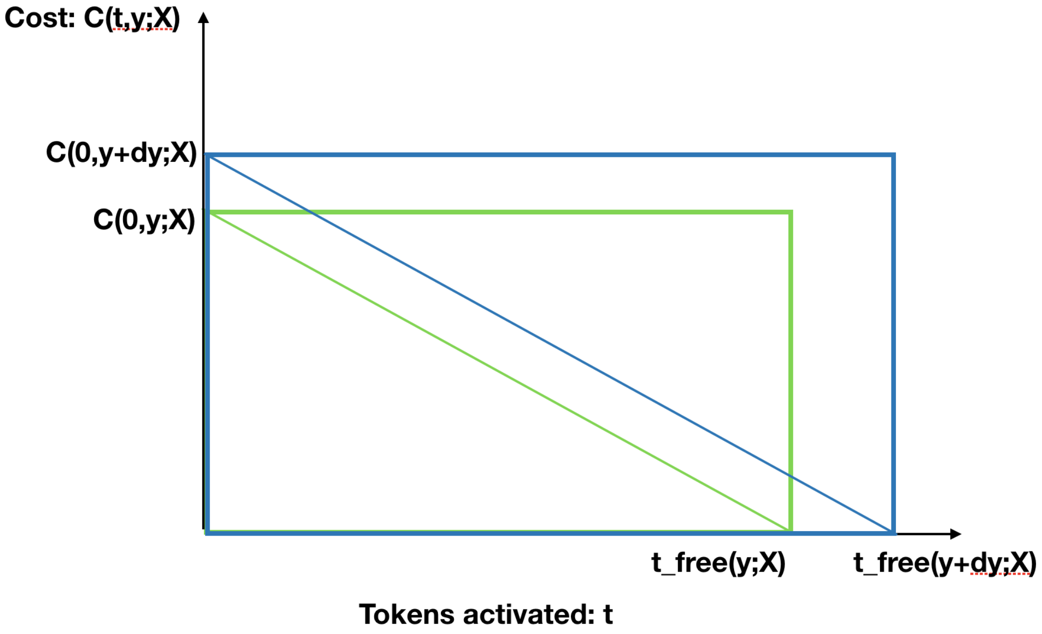

Figure 2.2: Relation of changes in t to changes in y

Property 1. *Observe that the utility per token per period of time can be derived from the cost savings with respect to tokens activated during that period:*

$$\begin{aligned} U(X) &= -\frac{\partial}{\partial t}C(t;y;X) && (2.15)\\ &= c\cdot y\frac{\partial}{\partial t}f(t;y;X) && (2.16)\\ &= \frac{c}{t_{free}(1;X)}, && (2.17) \end{aligned}$$

notably independent of the variables t and y. It immediately follows that the value of the token does not depend directly on the number of tokens being activated, but rather is proportional to the price set for the service and inversely proportional to $t_{free}(1;X)$. Now, it is possible to reason through an appropriate function $t_{free}(1;X)$.

2.5 Effects of the Network Growth

Discount tokens provide their holders access to the network and allow early users to benefit from the network effects generated as the product gains traction. Here, for every dollar spent by early participants, the discounts realized later in the network's lifetime grow if the network utilization grows.

A simple reuseable design is to define

$$t_{free}(1; X) = \beta \cdot \frac{X_T}{X_U} \tag{2.18}$$

where X_T is the total tokens activated networkwide, and X_U is the total number of units of service in use networkwide, each computed over the most recent period from the data on the blockchain. The parameter $\beta > 0$ is a proportionality coefficient, a system-level design parameter the meaning of which will be explained in section 2.7.

We see that this formulation provides for reduction in $t_{free}(1; X)$ when X_U increases. This ensures that if you bought tokens early when there were relatively few users, those same tokens would provide the same share of the discounts which, assuming increased network usage, would resolve to a larger absolute discount capacity. That additional capacity could be used by the same user, or transferred to others. The cost of actually using the service remains reliably bounded by c because a user could always choose to buy the service from the network without using discount tokens.

> Alice bought 100 BCM tokens at a price of $1 per BCM during the MVP stage of BlockChainMail, which allowed her to rent and trade costumes for free. A year later, it takes only 30 tokens to receive a free use license. She then gave 30 tokens to her friend Bob for his birthday (giving him a permanent free license to use BlockChainMail) and sold 40 tokens on a crypto-exchange at a price of $3 each for $120 total. She then continued to enjoy the services of BlockChainMail for free.

2.6 Deriving Value from Utility

Using the formulation in equation (2.18) with the utility definition in equation (2.17), the utility value per token activated per unit of time is simply

$$U(X) = \frac{c}{\beta} \cdot \frac{X_U}{X_T}. \tag{2.19}$$

Clearly, the utility of tokens is increasing in the number of active users X_U. Since this is a per-license-period utility, it is appropriate to consider the time value of these savings. Such value can be derived by assuming that the user chose to buy and use the discount tokens as opposed to holding a cryptoasset that is providing a rate of return r. Define the fair value of the token as $\bar{U}(X)$ according to the present value of a perpetual annuity, with the simplifying assumption that adoption and thus X has reached steady state, making $U(X)$ constant,

$$\bar{U}(X) = \frac{1}{r} U(X) = \frac{c\, X_U}{r\beta\, X_T}. \tag{2.20}$$

Since not all services are time-based, this model requires a time component to prevent users from extracting infinite value from their tokens with a quick turnaround. We assume that activated tokens are locked for a specified time period, allowing one to apply the time-discount pricing model to these tokens in the general case. For use cases that are not otherwise time-based, one can define an activation period Δ, and y can be defined as level of service per period Δ, which is sufficient in most cases to avoid the degenerate case described.[1]

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[1] There are many ways to define fair value of discount tokens. For an alternative pricing model that uses commercial real estate as a baseline asset, see [31].

Let's say that a BlockChainMail software license costs \$10 per month; that BCM are activated for a minimum duration of a whole month with the effective discount fixed for the license period based on the state of the network at the time of the transaction. For simplicity in this example $\beta = 1$.

Set $c = \$10$ and $r = 14\%$. The rate of return r is selected based on the median 30-day rate of return for holding ether over any 30-day period between July 2015 and September 2017. Then, computing X_U by counting the number of unique user accounts with an active license, regardless of the fees paid, and X_T computed by summing over all quantities of BCM tokens activated in license contracts, up to their respective t_{max}.

$$\bar{U}(X_U, X_T) = 71.43 \cdot \frac{X_U}{X_T} \tag{2.21}$$

and if we assume there are 1,000 active users and 10,000 BCM tokens escrowed as software licenses, then there is an implied value of the BCM tokens

$$\bar{U}(1000, 10000) = \$7.14 \tag{2.22}$$

with a clear relationship that as the demand for the software exceeds the supply of BCM actively in use, the utility of those tokens rises to support that demand:

$$\bar{U}(3000, 15000) = \$14.29; \tag{2.23}$$

the number of active users X_U tripled to $3,000$, but the number of tokens activated only rose by 50% to $X_T = 15,000$ causing the utility of the token to double.

Under the token model described here, the number of tokens needed to access the same service fell by a factor of two, allowing twice as many users to access the software with the same number of tokens. Users are incentivized to move from using fiat to using discount tokens because the same tokens can be activated each period for perpetual use, and extra tokens can be resold on the open market.

2.7 Considering Operating Costs

In practice, there is a non-zero cost to maintaining and running a well-functioning service. If our example service, BlockChainMail, is developed using Ethereum, then there are costs to every transaction processed by its network. Additionally, there are maintenance and operational costs to the service, including front-end development, administrative costs, and other significant expenses.

More generally, this case demonstrates that the discount token model needs to account for funds required for the ongoing operation and maintenance of the network. One way to achieve this is to limit the number of tokens activated t by some value t_{max} strictly less than t_{free}.

A more creative approach is to modify the proposed discount function $f(t, y; X)$ to prevent *global* fee elimination, while still making it possible for individual users to receive services for free. The introduction of the parameter β in equation (2.18) makes this possible. We now prove that the total amount of fees that can be eliminated in discount token economics will not exceed $\frac{1}{\beta}$, ensuring that there is always sufficient cashflow to operate the network.

Property 2. *The networkwide revenue is given by*

$$\sum_{i=1}^{n} C(t_i, y_i; X) = c \cdot \left(1 - \frac{1}{\beta}\right) \sum_{i=1}^{n} y_i \tag{2.24}$$

associated with the global discount fraction $\frac{1}{\beta}$*,*

$$\begin{aligned} C(0, \sum_{i=1}^{n} y_i; X) - \sum_{i=1}^{n} C(t_i, y_i; X) &= \frac{c}{\beta} \sum_{i=1}^{n} y_i & (2.25) \\ &= \frac{1}{\beta} C(0, \sum_{i=1}^{n} y_i; X) & (2.26) \end{aligned}$$

with the following definitions for the network state variables:

$$X_T = \sum_{i=1}^{n} t_i \tag{2.27}$$

and

$$X_U = \sum_{i=1}^{n} y_i. \tag{2.28}$$

This may seem complex, but it is actually a network property inherited from the definition of t_{free} found in equation (2.18). If one were to choose $\beta = 1$, a very interesting property would arise: for any total usage and total tokens activated (the values of which are strictly positive), the tokens would be sufficient to offset all of the costs, as long as they are distributed in proportion to each user's utilization of the network. Consequently, the network could potentially eliminate all revenue, and thus prevent funding of software maintenance and operations.

In order to address these concerns, the additional requirement that $\beta > 1$ is introduced. From property 2, it is clear that $1/\beta$ provides the upper limit on discounts globally. (In practice, given the trivial usability requirement that discounts are locked at a constant value for the entire time period, additional constraints may be necessary early in the network's lifetime.) This valuable property is ensured at the network level, despite the freedom of individual agents to decide their level of service y and their tokens activated t. This is only possible due to the *state feedback* construction where t_{free} is a function of the network state X.

This is part of the core innovation of the discount token model.

In blockchain-based systems, consensus is globally maintained over the network state, and the smart contract implementing discount token functionality can dynamically adjust to changes in the global state X. This approach leverages a rudimentary case of *state feedback control* to create a network level dynamic system with a stable equilibrium at the desired revenue splitting ratio, $1/\beta$ for the users and $1 - 1/\beta$ for the ongoing operational expenses. This paper presents the steady state property for the general case of a discount token. An analysis of a particular instance of a discount token would allow for detailed consideration of the transient dynamics; for a more detailed treatment of state feedback control, see [4].

2.8 Economic Components of Token Value

Whether the purchase was initially speculative or made with intent to utilize the discounts, any holder of a discount token is strictly better off using the token, $V_K^{(user)} > V_K^{(investor)}$. Without loss of generality, the time of purchase is denoted $k = 0$, and the time of sale is denoted $k = K$, where the discrete time intervals are the service periods Δ. The token holder may choose to use the token for themselves or escrow it on behalf of others.

Property 3. *Purchasing a quantity m of the discount at time $k = 0$ and selling it at time $k = K$, the investor realizes value*

$$V_K^{(investor)} = m \cdot \left(P_K^{(token)} - P_0^{(token)}\right) \tag{2.29}$$

where $P_k^{(token)}$ is the market price at time k. A user of the token over the same time period realizes

$$V_K^{(user)} = m \cdot \left(P_K^{(token)} - P_0^{(token)}\right) + m \cdot \sum_{k=0}^{K} U(X_k) \tag{2.30}$$

where $U(X_k)$ is the utility at period k and all m tokens are fully utilized at each time k.

While the value realized by the investor may be positive or negative depending on the price moves of the token over an arbitrary interval defined as $k = 0$ to $k = K$, the utility value $U(X_k)$ is strictly positive for all k. The positivity of U_k can be verified from its definition in equation (2.19), as it is a simple ratio containing only strictly positive coefficients, c and β, and strictly positive network states X_U and X_T. It follows that the value from use is not only additive with the value from the investment, but that due to the positivity of the utility $U(X)$, the token is strictly more valuable to an investor who is a user than to one who is not. This is expected to manifest itself as a steady flow of tokens ownership from pure investors to investor users to the benefit of both sellers and buyers.

Figure 2.3 shows the ROI of a user holding tokens as a function of network growth. The yellow line indicates an optimal number of tokens for a given utilization of the service. If a user keeps the tokens in

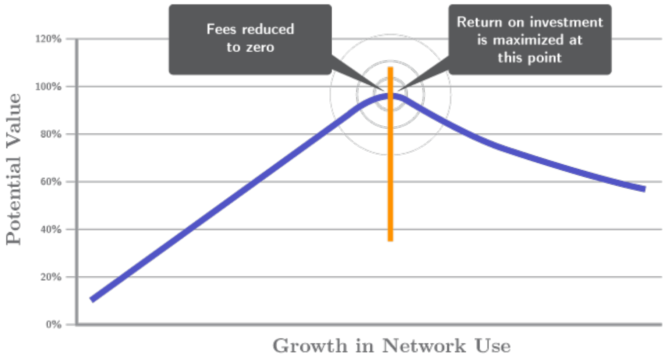

Figure 2.3: Return on investment for discount tokens based on network growth.

excess of what they can utilize, their return on investment will diminish, since they cannot realize the full value of the investment.

Alice bought 400 BCM tokens at a price of $1 per BCM during the MVP stage of BlockChainMail, which allowed her to rent and trade costumes for free. She used the tokens for five years before leaving the platform and selling them. Assuming the token price rose modestly to $1.50 per BCM, Alice realized an internal rate of return (IRR) of 35%.

Jill speculated by buying and selling 400 BCM tokens at the same times as Alice. Her gain of $200 constitutes a modest return over five years equal to an IRR of merely 8%. See figure 2.4.

	Purchase	Discount from Use Year 1	Discount from Use Year 2	Discount from Use Year 3	Discount from Use Year 4	Discount from Use Year 5	Sale at End of 5 Years	IRR
User	($400)	$120	$120	$120	$120	$120	$600	35%
Investor	($400)	$0	$0	$0	$0	$0	$600	8%

Figure 2.4: Example of using BCM over 5 years, rather than simply speculating.

3 Broader Implications

We can look at $U(X)$ as the price at which users would trivially decide to buy tokens as a license for one month of use. $\bar{U}(X)$ (defined in equation 2.20), on the other hand, captures the incentives around the long-term view, where users lock the same tokens over and over to continue accessing the software and expect to sell them again if they ever stop using them.

If the user base grows, the demand for tokens will drive the fiat value of tokens up without increasing the actual price of the services. The result is that early adopters not only have access to your service, but actually have the power to give or sell the service to others, effectively making them co-owners of the network. The model explicitly shares the network effect value with the early investors, but what they get is not money but increased access to the service.

It is very important to emphasize that the full value of the discount can only be realized by using the product. This is in stark contrast with gift cards, currencies, or coupons, the value of which is the same to both customers and passive holders. Noting that discount tokens possess both a use value and a resale value, with the former only accessible to active customers, we expect that passive holders and speculators will always be disinclined to hold these assets.

Active users receive some of the value that passive speculators are unable to realize. Indeed, a token is only worth to the speculator as much as it is worth to the user to whom such speculator will sell the token in the future. The longer the passive speculators hold the token, the more the users benefit (1) from tokens not being active in the system, and (2) from the price arbitrage between the speculator value and the user value when the tokens are finally sold.

Thus, discount tokens are similar to other investible property oriented to use, rather than passive ownership. One such type of property is residential real estate, which often shows similar dynamics: the owner of an apartment will receive the highest value of ownership when they live there, while a passive landlord has to find other ways to exercise such value, for example through rentals. Passive ownership of apartments you can't rent out is a really bad strategy. Similarly, a taxi medallion (such as those issued to New York City yellow cabs) is an investible license that generates the largest value only if utilized.

These examples demonstrate the difference between the discount tokens and securities in that the former require active utilization to generate the most value. One could also say that they are *the opposite* of a Ponzi scheme, because discount tokens channel value from early investors to later holders (customers) and not the other way around.

3.1 Relationship to Crowdsales

The discount token as a software license model is best coupled with a tranche token sale method, which releases tokens slowly over time, carefully accounting for network utilization data. The company developing and maintaining the network does not necessarily want to introduce all of the licenses at day one. Holding back the tokens increases their early market value. By retaining a large share of the supply and introducing it into circulation slowly, the company or foundation can control the health of their economy. To ensure that the token supply is never totally released, a convergent drip model should be used; this model is outlined in the Sweetbridge Liquidity Protocol Mathematical Specification, [31].

As with the discount token, the convergent drip model is useful to the community as a form of best practice and will be explicitly supported by the Sweetbridge Crowdsale Platform, [18]. In brief, it requires that a share of the fixed supply of tokens be allocated for sale, and that small fractions of the remaining share are released at prices well-supported by the fair value at the time of the release, and accounting for the growth of network. A subsequent document will expand on the use of this best practice.

4 Foundations of Interdisciplinary Research

This presentation draws on an extensive and interdisciplinary body of academic research. Motivated by the requirements stated in the introduction, the authors have leveraged the study of game theory, [21], both cooperative [25, 10, 5] and non-cooperative [17, 7]. Game theory is applied with a focus on stability around desirable economic equilibria [20, 2] using mathematical tools from control systems engineering, [23, 30]. While game theory establishes a basis for engineering systems involving human actors, the basic assumptions of rationality and consistency are often violated, [29, 27, 28]. Research then falls to the more empirical subfields of psychology and economics, which establish a well rounded understanding of such systems with human actors in the loop. Socio-technical networks, or the social graphs which exist through human interactions via technology, are further explored in [19] and [12].

Once one has moved beyond rationality assumptions it is important to bear in mind formal notions of fairness [11, 14, 24] in the study of social choice, [3]. When engineering systems that influence incentives while allowing freedom of choice one must consider the costs to the agent of acting against their incentives and impact such actions have on the system at large, [6, 2]. Control Systems Engineering [16, 22] allows for the enforcement of desirable network level invariant properties even when choice is retained at the agent level; this requires the observation of mathematical equivalencies between engineered networks and markets through input-output systems, [1] and duality theory, [15, 8]. Multi-agent systems in an engineering context have information asymmetries leading to actions misaligned to global objectives, so this field provides valuable formal tools [13, 26, 9] even when individual actors can be expected to deviate from the behaviors that are strictly aligned with their financial incentives.

The authors hope that the research foundations referenced here provide others the necessary tools to define and analyze their token models in a more rigorous way. As with the discount token archetype, the aim is to further foster responsible decentralized product development efforts, clarify regulatory concerns, drive greater transparency, and incentive alignment among stakeholders.

5 Acknowledgements

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We felt that the discount token model was an innovation that extended well beyond Sweetbridge and chose to extract and present a more general formulation of the idea to serve as a best practice in the community. Projects launching software products within the Ethereum ecosystem who are interested to join the Sweetbridge stable currency ecosystem and the project alliance so as to make use of the Sweetbridge Crowdsale Platform are encouraged to contact Sweetbridge for more information.

About Sweetbridge

Sweetbridge sponsors the development of blockchain-based economic protocols and applications to transform high-friction global supply chains into Liquid Value Networks. The Sweetbridge Alliance is an independent member-run non-profit that is building a global network of organizations comprised of interested industry technologists, blockchain projects, and open-source contributors from around the world. Its goal is to transform brittle, industrial-era commerce through decentralized industry ecosystems that create a faster, fairer value exchange, unleash working capital, better utilize resources, and optimize talent for the benefit of all participants. Follow us on Twitter at @sweetbridgeinc.

About BlockScience

BlockScience is a technology research and analytics firm specializing in the design and evaluation of decentralized economic systems. We are defining and practicing the emerging field of *Economic Systems Engineering* by applying the mathematical engineering technologies associated with the decades-old *Systems Engineering* field, along with *Game Theory* and *Behavioral Economics* fields, to the economic networks being instantiated via blockchain and smart-contract-enabled applications. Our work includes pre-launch design and evaluation based on real analysis and simulation, as well as post-launch monitoring and maintenance via reporting, analytics and decision support software development promoting economic health KPIs.

About CoinFund

CoinFund is a blockchain technology research company, advisory team, and private cryptoasset-focused investment vehicle. We work with companies to help apply blockchain technology to a variety of real-world problems. CoinFund promotes the culture of responsibility and stakeholder alignment in funding of decentralized organizations and business models.

6 Appendix: Proof of Property 2

Proof. Consider a set of n active user actions i composed of a level of service y_i and a number of tokens activated t_i, without making assumptions about the network state X_i, the network state when action i occurred. The current state defined

$$X = (X_T, X_U) = \left(\sum_{i=1}^{n} t_i, \sum_{i=1}^{n} y_i\right) \tag{6.1}$$

can be used for any time without loss of generality. Applying equation (2.14) across all active use

$$\begin{aligned}
\sum_{i=1}^{n} C(t_i, y_i; X) &= c \cdot \sum_{i=1}^{n} \frac{y_i \cdot t_{free}(1; X) - t_i}{t_{free}(1; X)} && (6.2)\\
&= c \cdot \left(\sum_{i=1}^{n} y_i - \sum_{i=1}^{n} \frac{t_i}{t_{free}(1; X)}\right) && (6.3)\\
&= c \cdot \left(X_U - \frac{X_T}{t_{free}(1; X)}\right). && (6.4)
\end{aligned}$$

Applying equation (2.18), the expression further simplifies to

$$\begin{aligned}
\sum_{i=1}^{n} C(t_i, y_i; X) &= c \cdot \left(X_U - X_T \cdot \frac{X_U}{\beta \cdot X_T}\right) && (6.5)\\
&= c \cdot X_U \cdot \left(1 - \frac{1}{\beta}\right), && (6.6)
\end{aligned}$$

demonstrating that in fact the total cost being paid is always $c \cdot \left(1 - \frac{1}{\beta}\right)$ times the total utilization X_U regardless of when that use occurred and how many tokens were activated with respect to that specific action. Our stated property holds, because applying equation (2.5),

$$C(0, \sum_{i=1}^{n} y_i; X) = c \cdot \sum_{i=1}^{n} y_i \tag{6.7}$$

for any X, allowing computation of the total discount as

$$\begin{aligned}
c \cdot X_U - \sum_{i=1}^{n} C(t_i, y_i; X) &= c \cdot X_U - c \cdot X_U \cdot \left(1 - \frac{1}{\beta}\right) && (6.8)\\
&= c \cdot \frac{X_U}{\beta} && (6.9)\\
&= \frac{c}{\beta} \sum_{i=1}^{n} y_i && (6.10)
\end{aligned}$$

by applying the definition of X_U found in equation (2.28). $\square$

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